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18.01 Single Variable Calculus
Fall 2006

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Practice Question For Exam 4

$$1) \frac{x-4}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} = \frac{(A+B)x^2 + (B+C)x + 4A+C}{(x+1)(x^2+4)} + \int \frac{x}{x^2+4} dx$$

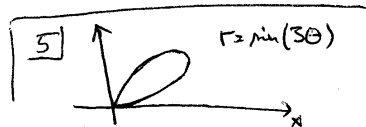
$$\begin{cases} A+B=0 \\ B+C=1 \\ 4A+C=-4 \end{cases} \Rightarrow \begin{cases} A=-1 \\ B=1 \\ C=0 \end{cases} \quad \int \frac{x-4}{(x+1)(x^2+4)} dx = \int \left[\frac{-1}{x+1} + \frac{x}{x^2+4} \right] dx = -\ln|x+1| + \frac{1}{2} \ln|x^2+4| + C$$

$$2) \int \frac{2x}{(x^2+4)^2} dx = \int_0^{\pi/4} \frac{2 \sec^2 u}{(4 \tan^2 u + 4)^2} du = \frac{1}{8} \int_0^{\pi/4} \cos^2 u du = \frac{1}{8} \int_0^{\pi/4} (1 + \cos(2u)) du = \frac{1}{16} \left(u + \frac{1}{2} \sin(2u) \right) \Big|_0^{\pi/4} = \frac{1}{64} (\pi + 2)$$

$$3) \int \frac{x^{2n}}{x^{2n+1}} \frac{e^{-x^2}}{x} dx = \frac{1}{2n+1} x^{2n+1} e^{-x^2} \Big|_0^1 - \int_0^1 (-2x e^{-x^2}) \frac{1}{2n+1} x^{2n+1} dx$$

$$n=k-1 \quad \int_0^1 x^{2k-2} e^{-x^2} dx = \frac{1}{2k-1} \cdot \frac{1}{e} + \frac{2}{2k-1} \int_0^1 x^{2k} e^{-x^2} dx$$

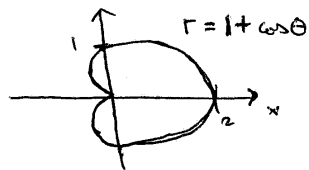
$$\int_0^1 x^{2k} e^{-x^2} dx = \frac{-1}{2e} + \frac{2k-1}{2} \int_0^1 x^{2k-2} e^{-x^2} dx$$



$$A = \int_0^{\pi/3} \frac{1}{2} \sin^2(3\theta) d\theta = \frac{1}{4} \int_0^{\pi/3} (1 - \cos(6\theta)) d\theta = \frac{\theta}{4} - \frac{1}{24} \sin(6\theta) \Big|_0^{\pi/3} = \frac{\pi}{12}$$

$$8) M = \int_0^a 8 \cdot 2\pi r dr = 2\pi \int_0^a r^2 dr = \frac{\pi}{2} r^3 \Big|_0^a = \frac{\pi}{2} a^3$$

9) 4a



$$\begin{aligned} r &= 3 \cos \theta \\ r &= 1 + \cos \theta \\ 3 \cos \theta &= 1 + \cos \theta \\ \cos \theta &= 1/2 \end{aligned}$$

$$\left(\theta = \frac{\pi}{3}, r = \frac{3}{2} \right) \\ \left(\theta = \frac{5\pi}{3}, r = \frac{3}{2} \right)$$

$$\begin{aligned} \text{ii. } (x - \frac{3}{2})^2 + y^2 &= (\frac{3}{2})^2 \\ x^2 - 3x + y^2 &= 0 \\ r^2 \cos^2 \theta - 3r \cos \theta + r^2 \sin^2 \theta &= 0 \\ r^2 - 3r \cos \theta &= 0 \\ r &= 3 \cos \theta \end{aligned}$$

$$6) a) L = \int ds = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^{\pi/2} \sqrt{(-3 \cos^2 t \sin t)^2 + (3 \sin^2 t \cos t)^2} dt = 3 \int_0^{\pi/2} \cos t \sin t (\cos^2 t + \sin^2 t)^{1/2} dt = \frac{3}{2} \int_0^{\pi/2} \sin(2t) dt = -\frac{3}{4} \cos(2t) \Big|_0^{\pi/2} = \frac{3}{2}$$

$$b) A = \int 2\pi y ds = \int_0^{\pi/2} 2\pi \sin^3 t \left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \right)^{1/2} dt = 2\pi \int_0^{\pi/2} \sin^3 t \cdot 3 \cos t \sin t dt = 6\pi \int_0^{\pi/2} \sin^4 t \cos t dt$$

$u = \sin t$
 $du = \cos t dt$

$$= 6\pi \int_0^1 u^4 du = \frac{6\pi}{5}$$

$$7) L = \int_0^{\pi} \sqrt{1 + \cos^2 x} dx$$

$$\cos^2 x \leq 1, \text{ so } \sqrt{1 + \cos^2 x} \leq \sqrt{2}$$

$$\text{so } L = \int_0^{\pi} \sqrt{1 + \cos^2 x} \leq \pi \sqrt{2}$$

9) i. circle of radius $\frac{3}{2}$ centered at origin:

$$r = \frac{3}{2}, \quad \frac{3}{2} = 1 + \cos \theta, \quad \cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3} \quad \left(\theta = \frac{\pi}{3}, r = \frac{3}{2} \right); \left(\theta = \frac{5\pi}{3}, r = \frac{3}{2} \right)$$

$$r = \frac{3}{2}$$